

# Calculus (High School Math)

## Course Curriculum (MPSL Academy)

**Each Session: 1 hour 15 minutes**

(First 10 min review of last week topics/HW, 50 min new topics, 15 min critical thinking Qs)

### 1. Sets, Functions, Graphs and Limits

- Elementary Set Theory
- Subsets
- Set Operations
- Coordinate Systems
- Distance Between Two Points in the Plane
- Graphs and Functions
- Increasing and Decreasing Functions
- Linear Dependence and Independence
- Single-valued Functions
- Parametric Representation of Curve
- Equation of Circle
- Types of Functions
- The Exponential and Logarithmic Functions
- The Trigonometric Functions
- Graphs of Trigonometric Functions
- The Hyperbolic Functions
- Symmetry of Functions
- Translation and Scaling of Axes
- Inverse Functions
- Equations of Lines
- Perpendicular Lines
- Limits
- Infinitesimals
- Limiting Value of a Function
- Formal Definition of a Limit

- Special Considerations
- Properties of Limits
- The Squeeze Theorem
- Continuous Functions and Discontinuous Functions
- Asymptotic Lines
- Finding Asymptotic Lines
- Conic Sections
- Circle
- Parabola
- Ellipse
- Hyperbola
- Conic Sections in Polar Coordinates
- Rotation of Axes
- General Equation of the Second Degree
- Computer Languages

## 2. Differential Calculus

- Slope of Tangent Line to Curve
- The Derivative of  $y = f(x)$
- Right and Left-hand Derivatives
- Alternative Notations for the Derivative
- Higher Derivatives
- Rules and Properties
- Differentiation of a Composite Function
- Differentials
- Differentiation of Implicit Functions
- Importance of Tangent Line and Derivative Function  $f'(x)$
- Rolle's Theorem
- The Mean-Value Theorem
- Cauchy's Generalized Mean-Value Theorem
- Derivative of the Logarithm Function
- Derivative of the Exponential Function

- Derivative and Continuity
- Maxima and Minima
- Concavity of Curve
- Comments on Local Maxima and Minima
- First Derivative Test
- Second Derivative Test
- Logarithmic Differentiation
- Differentiation of Inverse Functions
- Differentiation of Parametric Equations
- Differentiation of the Trigonometric Functions
- Simple Harmonic Motion
- L'Hôpital's Rule
- Differentiation of Inverse Trigonometric Functions
- Hyperbolic Functions and their Derivatives
- Approximations
- Hyperbolic Identities
- Euler's Formula
- Derivatives of the Hyperbolic Functions
- Inverse Hyperbolic Functions and their Derivatives
- Relations between Inverse Hyperbolic Functions
- Derivatives of the Inverse Hyperbolic Functions
- Table of Derivatives
- Table of Differentials
- Partial Derivatives
- Total Differential
- Notation
- Differential Operator
- Maxima and Minima for Functions of Two Variables
- Implicit Differentiation

### 3. Integral Calculus

- Summations
- Special Sums
- Integration
- Properties of the Integral Operator
- Notation
- Integration of derivatives
- Polynomials
- General Considerations
- Table of Integrals
- Trigonometric Substitutions
- Products of Sines and Cosines
- Special Trigonometric Integrals
- Method of Partial Fractions
- Sums and Differences of Squares
- Summary of Integrals
- Reduction Formula
- The Definite Integral
- Fundamental theorem of integral calculus
- Properties of the Definite Integral
- Solids of Revolution
- Slicing Method
- Integration by Parts
- Physical Interpretation
- Improper Integrals
- Integrals used to define Functions
- Arc Length, Area Polar Coordinates
- Arc Length in Polar Coordinates
- Surface of Revolution
- Mean Value Theorems for Integrals
- Proof of Mean Value Theorems
- Differentiation of Integrals
- Double Integrals

- Summations over nonrectangular regions
- Polar Coordinates
- Cylindrical Coordinates
- Spherical Coordinates
- Using Table of Integrals
- The Bliss Theorem

#### **4. Sequences, Summations and Products**

- Sequences
- Limit of a Sequence
- Convergence of a sequence
- Divergence of a sequence
- Relation between Sequences and Functions
- Establish Bounds for Sequences
- Additional Terminology Associated with Sequences
- Examples of Sequences
- Infinite Series
- Sequence of Partial Sums
- Convergence and Divergence of a Series
- Comparison of Two Series
- Test For Divergence
- Cauchy Convergence
- The Integral Test for Convergence
- Alternating Series Test
- Bracketing Terms of a Convergent Series
- Comparison Tests
- Ratio Comparison Test
- Absolute Convergence
- Slowly Converging or Slowly Diverging Series
- Certain Limits
- Power Series
- Operations with Power Series

- Maclaurin Series
- Taylor and Maclaurin Series
- Taylor Series for Functions of Two Variables
- Alternative Derivation of the Taylor Series
- Remainder Term for Taylor Series
- Modification of a Series
- Conditional Convergence
- Algebraic Operations with Series
- Bernoulli Numbers
- Euler Numbers
- Functions Defined by Series
- Generating Functions
- Functions Defined by Products
- Continued Fractions
- Terminology
- Evaluation of Continued Fractions
- Convergent Continued Fraction
- Regular Continued Fractions
- Euler's Theorem for Continued Fractions
- Gauss Representation for the Hypergeometric Function
- Representation of Functions
- Fourier Series
- Properties of the Fourier trigonometric series
- Fourier Series of Odd Functions
- Fourier Series of Even Functions

## **5. Applications of Calculus**

- Related Rates
- Newton's Laws
- Newton's Law of Gravitation
- Work, Energy, First Moments and Center of Gravity
- Centroid and Center of Mass

- Centroid of an Area
- Symmetry
- Centroids of composite shapes
- Centroid for Curve
- Higher Order Moments
- Moment of Inertia of an Area
- Moment of Inertia of a Solid
- Moment of Inertia of Composite Shapes
- Pressure, Chemical Kinetics, Rates of Reactions
- The Law of Mass Action
- Differential Equations
- Spring-mass System
- Simple Harmonic Motion,
- Damping Forces
- Mechanical Resonance
- Torsional Vibrations
- The simple pendulum
- Electrical Circuits
- Thermodynamics
- Radioactive Decay
- Economics
- Population Models
- Approximations
- Partial Differential Equations
- Easy to Solve Partial Differential Equations

**NOTE: STUDENTS WILL PARTICIPATE IN VARIOUS STATE & NATIONAL LEVEL MATH COMPETITIONS LIKE MATHCOUNTS, MATH OLYMPIAD, MATH KANGAROO, CONTINENTAL MATH LEAGUE AND SO ON. OUR GOAL IS TO BUILD CONFIDENCE IN MATH IN AN EFFICIENT WAY RESULTING IN IMPROVING SCORES IN SCHOOL EXAM & SUCCEED IN NATIONAL/INTERNATIONAL MATH COMPETITIONS.**